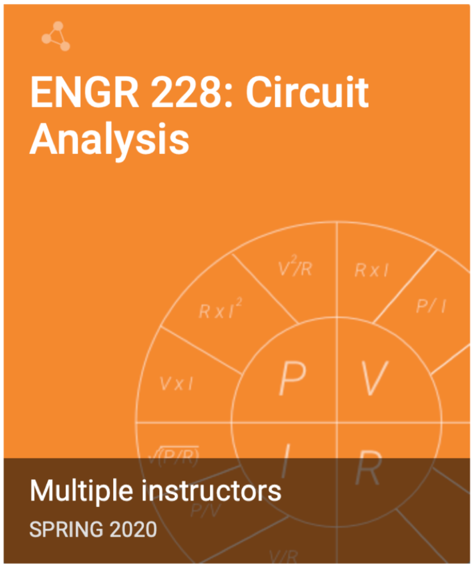


Chapter 5.6

Response of the RL circuit

Engr228 - Circuit Analysis

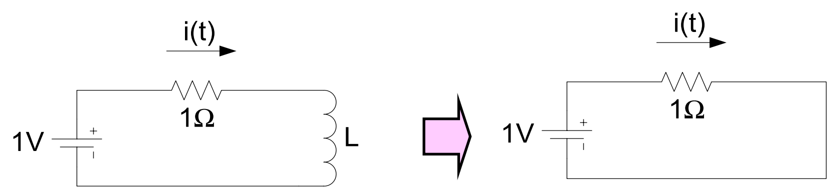
Spring 2020

Dr Curtis Nelson

Section 5.6 Objective

- Learn to:
  - Analyze the transient and steady-state responses of RL circuits.

Review: DC Characteristics of an Inductor



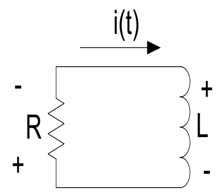
$$v_L(t) = L\frac{di_L(t)}{dt}$$

When $i_L(t)$ is constant, $di_L(t) = 0$ thus, $v_L(t) = 0$.
In other words, the inductor can be replaced with a short circuit.

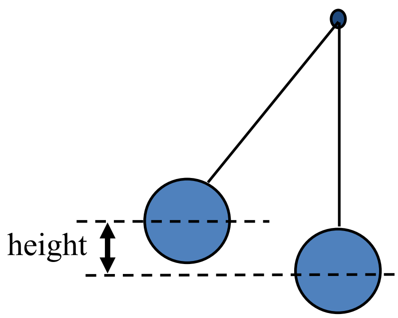
Types of First-Order Responses

- Transient response, natural response, homogeneous solution (temporary position change)
  - Fades to zero over time.
- Forced response, steady-state response, particular solution (permanent position change)
  - Follows the input;
  - Independent of time passed.

Source Free RL Circuits

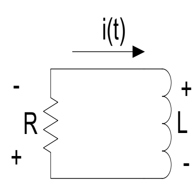


Inductor L has energy stored so initial current is I_0



Similar to a pendulum that is at a height h where the potential energy is nonzero.

Source Free RL Circuits



$$Ri(t) + L\frac{di(t)}{dt} = 0$$

$$\frac{di(t)}{dt} + \frac{R}{L}i(t) = 0$$

There are 2 ways to solve this first-order differential equation.

Solving Source Free RL Circuits

Method 1

Assume solution is of the form $i(t) = Ae^{st}$
where A and s are the constants that need to solved for.

Substitute $i(t) = Ae^{st}$ into the equation: $\frac{di(t)}{dt} + \frac{R}{L}i(t) = 0$

$$Ase^{st} + \frac{R}{L}Ae^{st} = 0$$

$$(s + \frac{R}{L})Ae^{st} = 0$$

$$s = -\frac{R}{L}$$

$$i(t) = Ae^{-\frac{R}{L}t}$$

Solving Source Free RL Circuits - continued

Initial condition: $i(0) = I_0$

from $i(t) = Ae^{-\frac{R}{L}t}$

$$I_0 = Ae^0$$

$$I_0 = A$$

Therefore $i(t) = I_0 e^{-\frac{R}{L}t}$

Solving Source Free RL Circuits

Method 2: Direct integration

$$\frac{di(t)}{dt} + \frac{R}{L} i(t) = 0$$

$$\frac{di(t)}{dt} = -\frac{R}{L} i(t)$$

$$\frac{di(t)}{i(t)} = -\frac{R}{L} dt$$

$$\int_{I_0}^{i(t)} \frac{di(t)}{i(t)} = \int_0^t -\frac{R}{L} dt$$

$$\ln i(t) \Big|_{I_0}^{i(t)} = -\frac{R}{L} t \Big|_0^t$$

$$\ln i(t) - \ln I_0 = -\frac{R}{L}(t-0)$$

$$i(t) = I_0 e^{-\frac{R}{L}t}$$

Time Constant

The ratio L/R is called the **time constant** and is denoted by the symbol τ (*tau*).

$$\tau = \frac{L}{R}$$ Units: seconds

One time constant is defined as the amount of time required for the output to go from its initial value I_0 to 36.8% of its initial value.

$$i(t) = I_0 e^{-\frac{R}{L}t} = I_0 e^{-\frac{t}{\tau}}$$

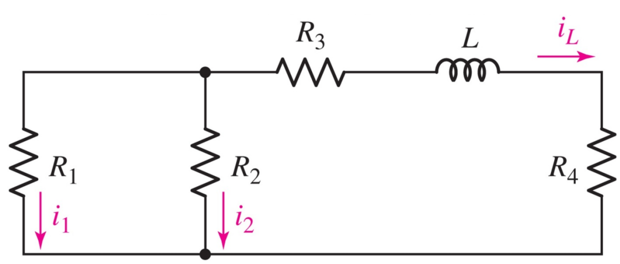
$$e^{-1} = 0.368$$

1^{st} Order Response Observations

- The current through an inductor or the voltage across a capacitor is the same *prior to* and *after* a switch at $t = 0$ seconds because these quantities cannot change instantaneously.
- ***All*** voltages and ***all*** currents in an RC or RL circuit follow the same natural response $e^{-t/\tau}$.

General RL Circuits

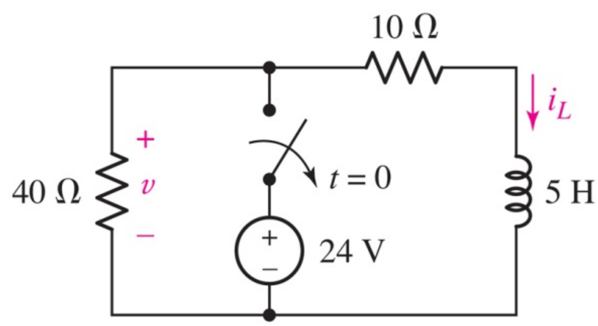
The time constant of a single inductor circuit will be $\tau = L/R_{eq}$ where R_{eq} is the resistance seen by the inductor.



Example: $R_{eq}=R_3+R_4+R_1R_2/(R_1+R_2)$

Example First-Order RL Circuit

Assume the switch is in the closed position for a long time for t < 0. Find the voltage *v(t)* at *t* = 200 mS.



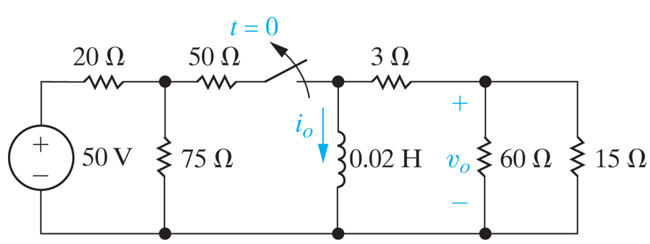
v(t) = -12.99 volts at t = 200 ms

Textbook Problem 7.7 (Nilsson 11th)

The switch is opened at t = 0 seconds.

a) Find $i_0(t)$ for t ≥ 0 seconds.

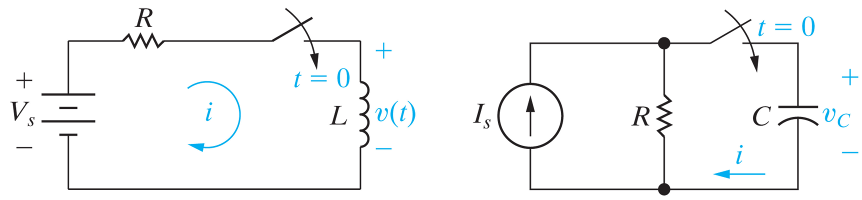
b) Find $v_0(t)$ for t ≥ 0^+ seconds.



$i_o(t) = 0.6e^{-750t}$ *Amps*

$v_0(t) = -7.20e^{-750t}$ *Volts*

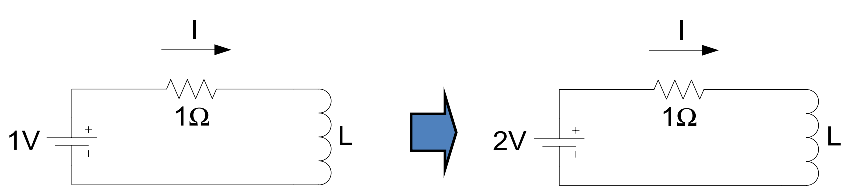
Driven RL and RC Circuits



- Many RL and RC circuits are driven by a DC or an AC source. The ***complete*** response of a driven RL or RC circuit is the sum of the transient response and the forced response:

$$i(t) = tran(t) + forced(t)$$

Circuit Example

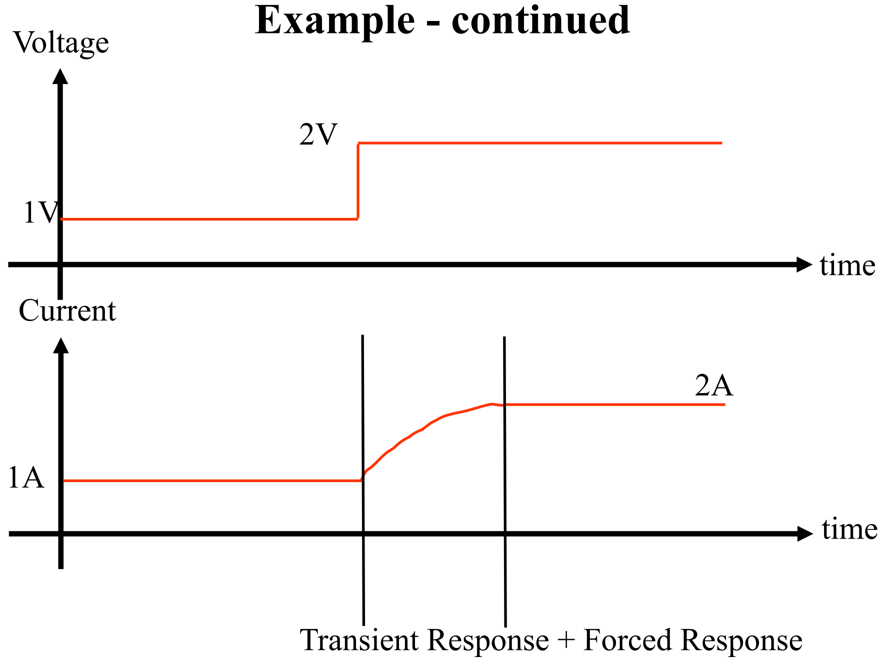


I = 1A

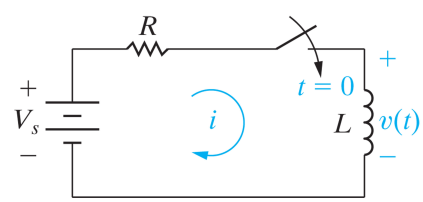
I = 2A

Suppose the voltage source changes abruptly from 1V to 2V.
Does the current change abruptly as well?

Example - continued



Complete Response



- The differential equation for the circuit above now becomes

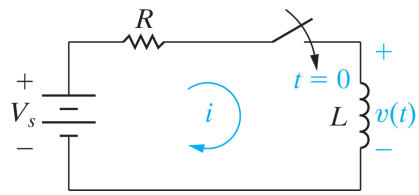
$$Ri(t) + L\frac{di(t)}{dt} = v_S(t)$$

- The transient part of the complete solution is determined by setting the forcing function v_S(t) = 0:

$$Ri(t) + L\frac{di(t)}{dt} = 0$$

Complete Solution for RL Circuits

The complete solution is found in section 5.6 of Zybooks:



$$i_L(t) = \frac{V_S}{R} + \left(i_L(0) - \frac{V_S}{R}\right) e^{-\left(\frac{R}{L}\right)t}$$

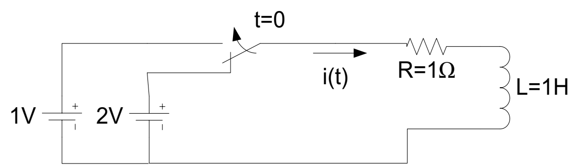
$$i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty))e^{-\left(\frac{R}{L}\right)t}$$

Procedure for Solving First-Order Circuits

1. Identify the variable of interest for the circuit. For RC circuits, it is most convenient to choose the capacitive voltage - for RL circuits, it is best to choose the inductor current.
2. Determine the initial value of the variable at $t = 0$. Note that if you choose capacitor voltage or inductive current, it is not necessary to distinguish between $t = 0^-$ and $t = 0^+$ because these variables are time-wise continuous.
3. Calculate the final value of the variable as $t \rightarrow \infty$.
4. Calculate the time constant of the circuit.

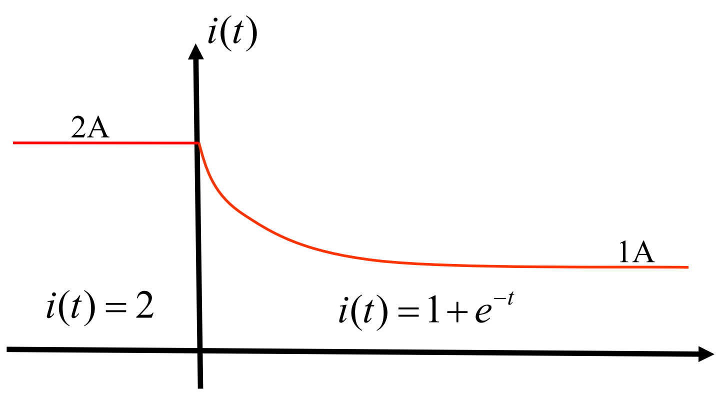
Example Problem

The switch is in the position shown for a long time before t = 0. Find i(t).



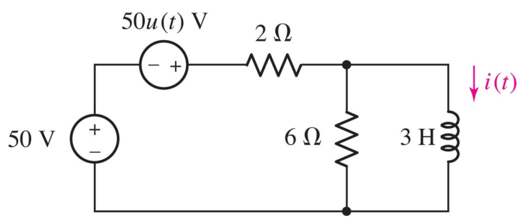
$$i(t) = 1 + e^{-t}$$

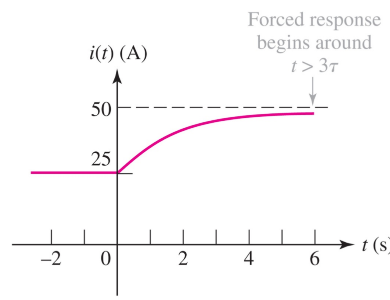
Example Graph



Example: RL Circuit with Step Input

Find *i(t)*.

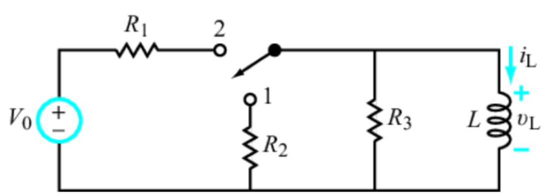




$i(t)=25+25(1-e^{-t/2})u(t)$ A

Example 5.6.1 Zybooks Total Response

After having been in position 1 for a long time, the switch is moved to position 2 at t = 0. Given that V_0=12V, R_1=30Ω, R_2=120Ω, R_3=60Ω, and L=0.2H, determine i_L(t) for all time.



$i_L(t) = 0.4(1 - e^{-100t})\ A$

Section 5.6 Summary

- You learned to:
  - Analyze the transient and steady-state responses of RL circuits.